21[L].—EARL D. RAINVILLE, *Special Functions*, The Macmillan Co., New York, 1960, xii + 365 p., 24 cm. Price $11.75.

This aptly titled, interesting, extremely well written book is based upon the lectures on Special Functions given by the author at the University of Michigan since 1946. The author's aim in writing the book was to facilitate the teaching of courses on the subject elsewhere. As an instructor in such a course, the reviewer feels certain that this most welcome text is to be accorded a warm reception on many college campuses.

More than fifty special functions receive varying degrees of attention; but, for the sake of usefulness, the subject is not approached on the encyclopedic level. Many of the standard concepts and methods which are useful in the detailed study of special functions are included. There is a great deal of emphasis on one of the author's favorite subjects: generating functions. Two interesting innovations are I. M. Sheffer's classification of polynomial sets and Sister M. Celine Fasenmyer's technique for obtaining recurrence relations for sets of polynomials. Functions of the hypergeometric family hold the center of the stage throughout a major portion of the text. The book concludes with a short current bibliography which should enable the reader to begin a more detailed study of the field.

There are twenty-one chapters in all, and the book may be roughly divided into four distinct parts:

(1) Two short preliminary chapters, 1 and 3, deal separately with infinite products and asymptotic series, respectively.

(2) Chapter 2 treats the gamma and beta functions, and chapters 4, 5, 6, and 7 are devoted to the hypergeometric family: the hypergeometric function, generalized hypergeometric functions, Bessel functions, and the confluent hypergeometric function, respectively.

(3) Chapter 8 is concerned with the generating function concept, as a preparation to chapters 9, 10, 11, and 12, which consider orthogonal, Legendre, Hermite, and Laguerre polynomials, respectively. Chapter 13 contains I. M. Sheffer's classification of polynomial sets; chapter 14 contains Sister M. Celine Fasenmyer's technique for obtaining recurrence relations for polynomials; and chapter 15 contains symbolic relations among classical polynomials. There follow three polynomial chapters: 16, 17, and 18, on Jacobi, ultraspherical and Gegenbauer, and other polynomials, respectively.

(4) The concluding three chapters, 19, 20, and 21, are devoted to elliptic functions, theta functions, and Jacobian elliptic functions, respectively.

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22[L].—L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge University Press, New York, 1960, x + 247 p., 29 cm. Price $12.50.

This is a valuable treatise on the subject, and the first of its kind in English. Tricomi [1] and Buchholz [2] have previously written books on the subject in Italian and German, respectively. Tricomi used the notation derived from the theory of hypergeometric functions, and this also is the principal notation employed in the Bateman Manuscript Project [3]. Buchholz uses the notation introduced by Whittaker. The notation of the present volume is a fusion of the two, and many results of the same character are given in all of the notations. This should prove most useful to research workers, as the confluen function has many applications, and notation is not uniform.

Chapter I studies the confluent hypergeometric functions as solutions of differential equations. Power series expansions are developed and relations between the various functions are carefully detailed.

Chapter II deals with differential properties including contiguous relations, Wronskians, addition theorems and multiplication theorems.

Chapter III is principally concerned with definite integrals involving the confluent functions. These include integrals of Barnes, Euler and Pochhammer types, and Laplace, Mellin and Hankel transforms.

Chapter IV takes up asymptotic expansions. It is a very useful compendium. In particular, as the author remarks, the theory is not complete, and it is in this area where important new results are anticipated.

Related functions such as Coulomb wave functions, Bessel functions, incomplete gamma functions, etc., are briefly considered in Chapter V.

Descriptive properties are the subject of Chapter VI. Zeros in x of ${}_1F_1(a; b; x)$ and $W_{k,m}(x)$, and zeros in a and b of the former are studied. Formulas and expansions for the zeros are provided. Numerical evaluation of ${}_1F_1$ is discussed and numerous figures are provided to illustrate its behavior when a, b, and x are real.

Three appendices give tables. These are as follows:

Appendix I. Smallest positive zeros of ${}_1F_1(a; b; x)$

$a = -4.0(0.1) - 0.1, \quad b = 0.1(0.1)2.5, \quad 7D$

Appendix II. ${}_1F_1(a; b; x)$

$a = -1.0(0.1)1.0, \quad b = 0.1(0.1)1.0, \quad x = 0.1(0.1)10.0,$

$7S - 9S$

Appendix III. ${}_1F_1(a; b; x)$

$a = -11.0(0.2)2.0, \quad b = -4.0(0.2)1.0, \quad x = 1, \quad 7D$

Appendix I was calculated to 8D on EDSAC, and the error does not exceed two units in the seventh decimal. Appendix II was calculated on the same machine, but no mention is made of accuracy. Some spot checks indicate that the error does not exceed four units in the last figure recorded. Appendices I and II are extensions of tables previously published by the author. Other short tables of the material in Appendix II have also appeared previously, but the present table is the most complete available. Appendix III was evolved using recurrence formulas and the data of Appendix II. It is quite surprising that nothing is said about interpolation.

A detailed table of contents, general index, and symbolic index of notation enhance the usefulness of the volume. The list of references, though not exhaustive, is fairly complete for work subsequent to that of Buchholz [2], whom the reader should consult for an extensive bibliography of work prior to about 1952. The reviewer notes that two important tables of Whittaker functions reviewed in [4] are not found in Miss Slater's list of references.

Y. L. L.

1. F. G. TRICOMI, *Lezioni sulle funzioni ipergeometriche confluenti*, Gheroni, Torino, 1952.
2. H. BUCHHOLZ, *Die konfluente hypergeometrische Funktion mit besonderer Berücksichtigung ihrer Anwendungen*, Springer, Berlin, 1953.
3. A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER, & F. G. TRICOMI, *Higher Transcendental Functions*, v. 1, Chapter VI, McGraw-Hill, New York, 1953.
4. RMT 46, *MTAC*, v. 12, 1958, pp. 86–88.

23[M, X].—G. DOETSCH, *Einfuhrung in Theorie und Anwendung der Laplace-Transformation*, Birkhäuser Verlag, Basel, Switzerland, 1958, 301 p., 24 cm. Price SFr 39.40.

This book forms an excellent introduction to the subject of Laplace transformations in one dimension, written by one of the leading experts in the field. From his wide knowledge of both the theoretical and applied aspects of the subject, the author has written a very readable, rigorous exposition which also indicates relations with appropriate physical concepts.

After a general introduction, the basic properties of the Laplace transform are developed in ten short chapters. Included are discussions of half-planes of convergence, uniqueness of the inverse, analytic properties of the transform, the effect of a linear transformation of the independent variable on the transform, the effect of differentiation and integration, and the transformation of a convolution. Because each chapter is devoted to a separate topic, the book is very useful for reference purposes. Many examples of specific transforms are given.

The next four chapters deal with the application of Laplace transforms to the following problems: the initial-value problem in ordinary differential equations with constant coefficients; the solution of differential equations for special input functions; homogeneous and non-homogeneous systems of differential equations; the initial-value problem for difference equations.

The next group of chapters deals with further properties of the transform, some of which may not be familiar to the reader: the behavior of the transform at infinity; inversion formulas expressed as integrals along vertical lines, as integrals along deformed paths in the complex plane, and as series of residues; conditions for the representation of a function as a transform; functions given as the sums of series of transforms; the analogue of Parseval's formula and transforms of products; and the asymptotic behavior of the transform and of the "original" function.

The book concludes with three chapters on further applications of the Laplace transform to differential equations with variable coefficients, to simple partial differential equations, and to certain integral equations.

A useful feature of the book is the inclusion of necessary background material, particularly in the later chapters.

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24[S, X].—G. I. MARČHUK, *Numerical Methods for Nuclear Reactor Calculations*, (An English translation of a work originally published in Russian as Supplement Nos. 3–4 of the Soviet Journal of Atomic Energy, Atomnaia Énergiia. Atomic Press, Moscow, 1958.) Consultants Bureau, Inc., New York, 1959, 293 p. 28 cm. Price $60.00.

A more accurate title for this book would have been *Numerical Methods for*